Post-quantum key exchange for the TLS protocol from the ring learning with errors problem

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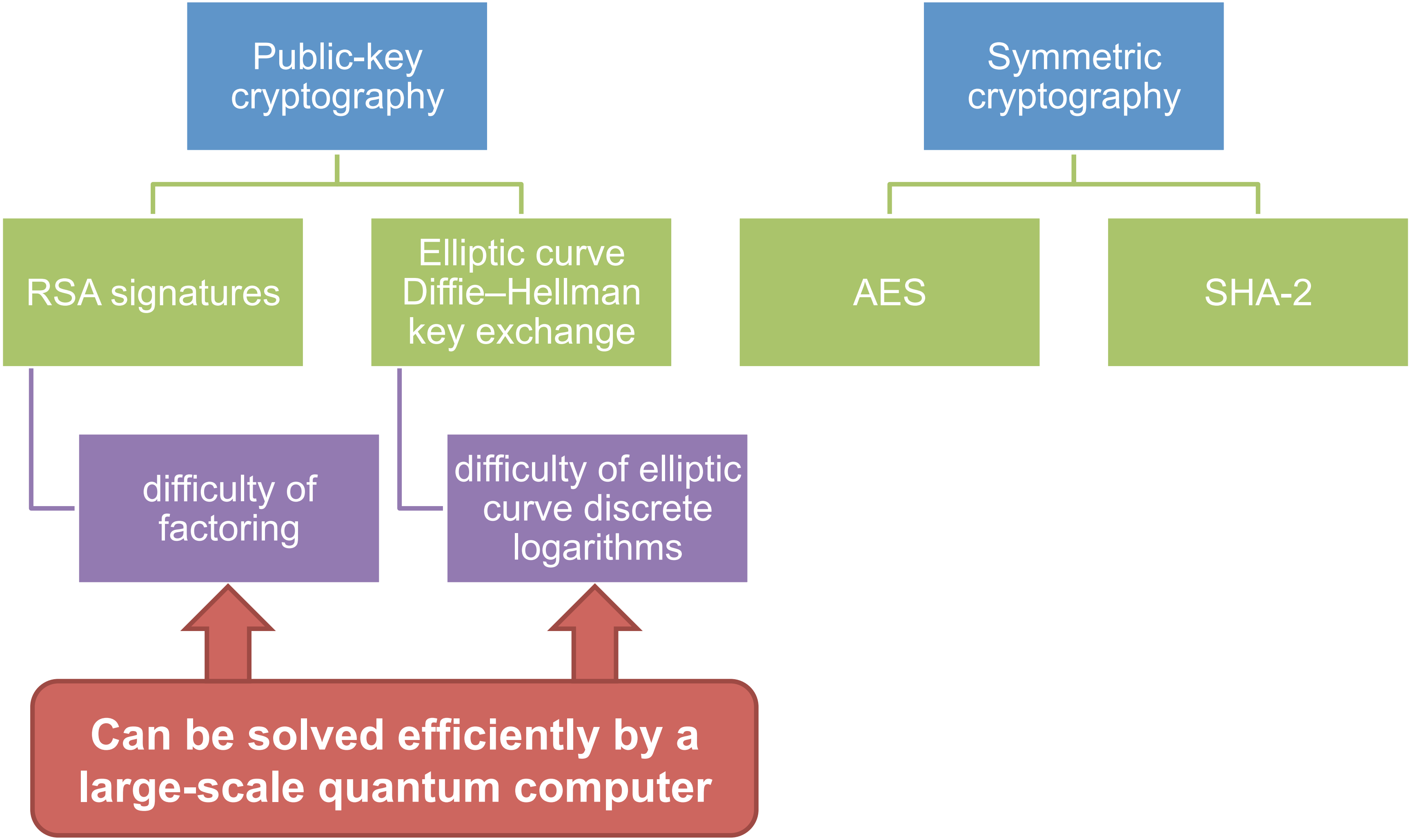
http://eprint.iacr.org/2014/599

Contemporary cryptography

TLS-ECDHE-RSA-AES128-GCM-SHA256



Public-key cryptography

Symmetric cryptography

RSA signatures

Elliptic curve Diffie–Hellman key exchange

AES

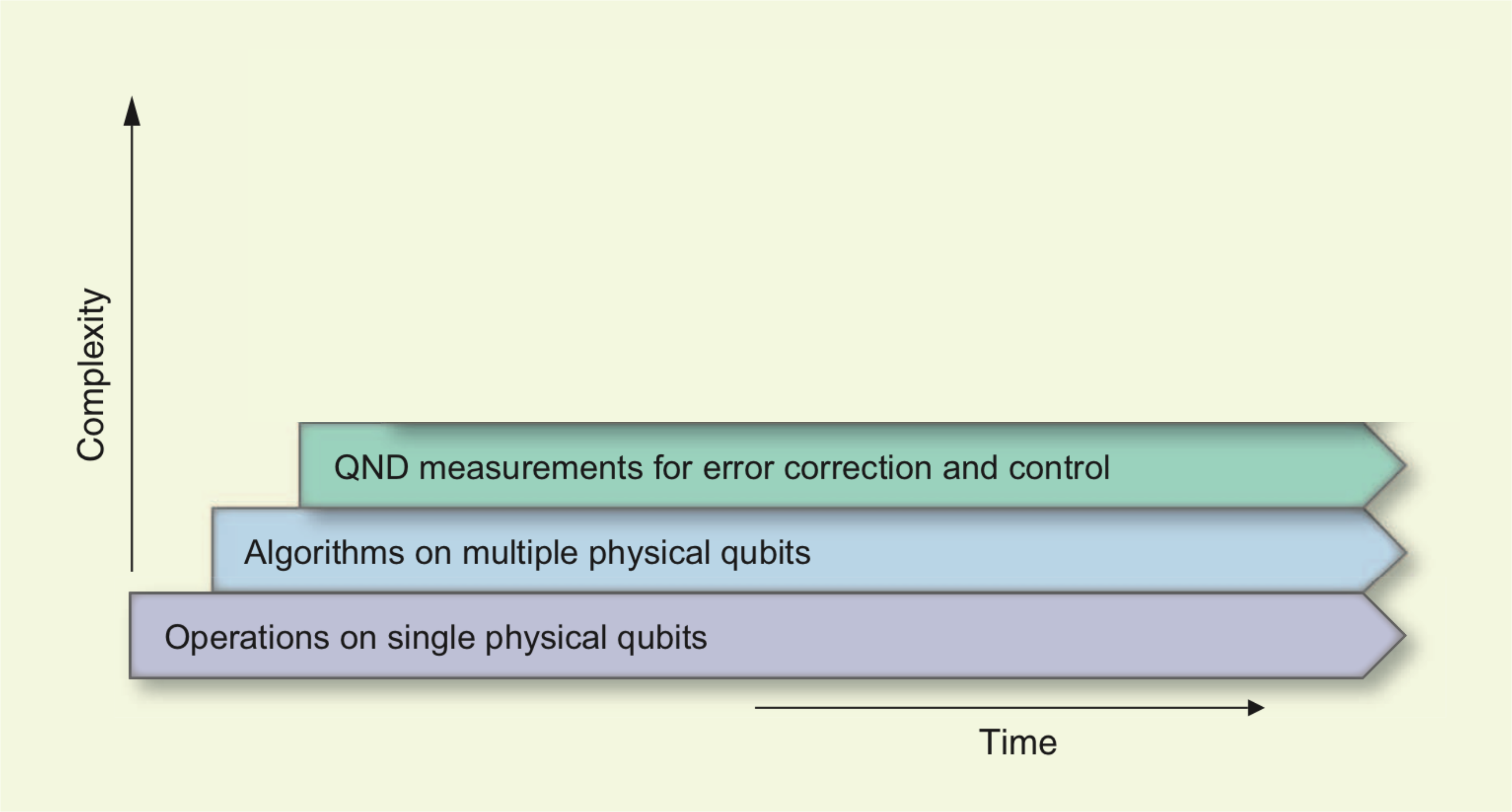
SHA-2

difficulty of factoring

difficulty of elliptic curve discrete logarithms

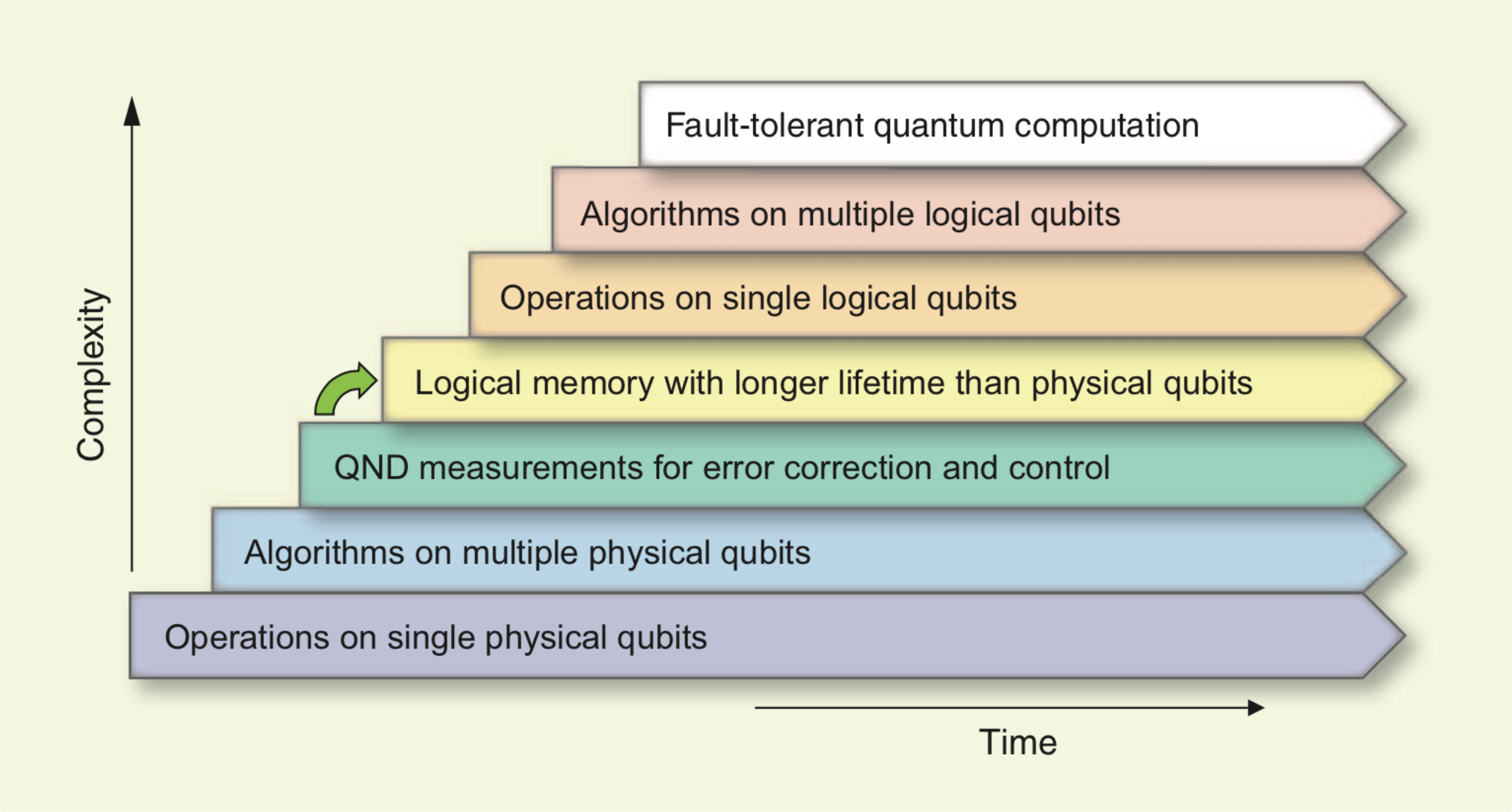
Can be solved efficiently by a large-scale quantum computer

Building quantum computers


Complexity
QND measurements for error correction and control
Algorithms on multiple physical qubits
Operations on single physical qubits
Time

Devoret, Schoelkopf. *Science* 339:1169–1174, March 2013.

Building quantum computers



Fault-tolerant quantum computation

Algorithms on multiple logical qubits

Operations on single logical qubits

Logical memory with longer lifetime than physical qubits

QND measurements for error correction and control

Algorithms on multiple physical qubits

Operations on single physical qubits

Devoret, Schoelkopf. *Science* 339:1169–1174, March 2013.

Post-quantum / quantum-safe crypto

No known exponential quantum speedup:

Code-based

- McEliece

Hash-based

- Merkle signatures
- Sphincs

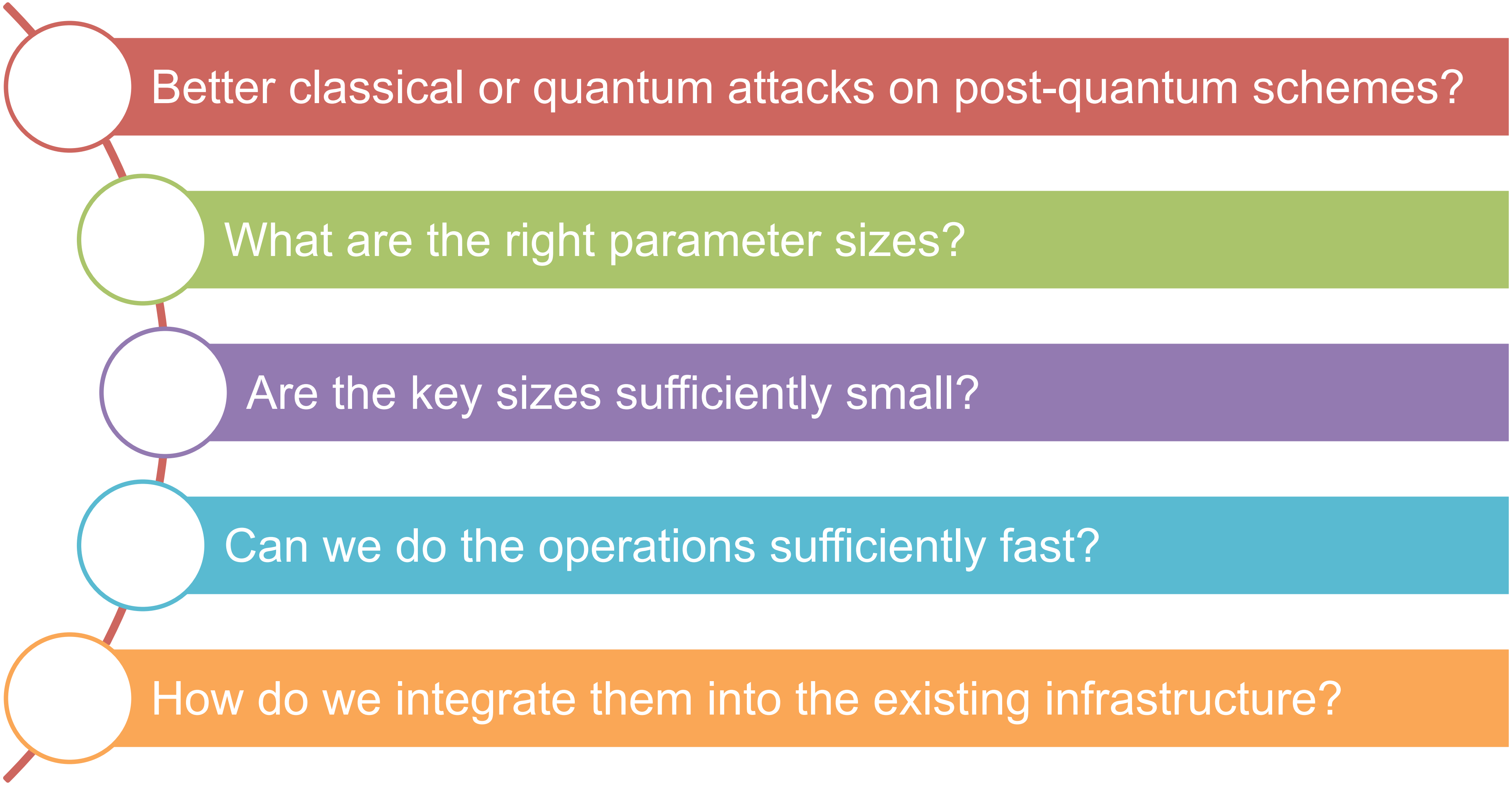
Multivariate

- multivariate quadratic

Lattice-based

- NTRU
- learning with errors
- ring-LWE

Lots of questions



Better classical or quantum attacks on post-quantum schemes?

What are the right parameter sizes?

Are the key sizes sufficiently small?

Can we do the operations sufficiently fast?

How do we integrate them into the existing infrastructure?

Lots of questions

This talk: ring learning with errors

- Are the key sizes sufficiently small?
- Can we do the operations sufficiently fast?
- How do we integrate them into the existing infrastructure?

This talk: ring-LWE key agreement in TLS

Premise: large-scale quantum computers don't exist right now, but we want to protect today's communications against tomorrow's adversary.

- Signatures still done with traditional primitives (RSA/ ECDSA)
  - we only need authentication to be secure *now*
  - benefit: use existing RSA-based PKI
- Key agreement done with ring-LWE

Solving systems of linear equations

$$\overset{\mathbb{Z}_{13}^{7\times 4}}{\begin{pmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{pmatrix}} \times \overset{\substack{\textbf{secret} \\ \mathbb{Z}_{13}^{4\times 1}}}{\begin{pmatrix} \\ \\ \\ \end{pmatrix}} = \overset{\mathbb{Z}_{13}^{7\times 1}}{\begin{pmatrix} 4 \\ 8 \\ 1 \\ 10 \\ 4 \\ 12 \\ 9 \end{pmatrix}}$$

Linear system problem: given **blue**, find **red**

Solving systems of linear equations

$$\begin{array}{ccccc} \mathbb{Z}_{13}^{7\times 4} & & \overset{\textbf{secret}}{\mathbb{Z}_{13}^{4\times 1}} & & \mathbb{Z}_{13}^{7\times 1} \\ \begin{pmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{pmatrix} & \times & \begin{pmatrix} 6 \\ 9 \\ 11 \\ 11 \end{pmatrix} & = & \begin{pmatrix} 4 \\ 8 \\ 1 \\ 10 \\ 4 \\ 12 \\ 9 \end{pmatrix} \end{array}$$

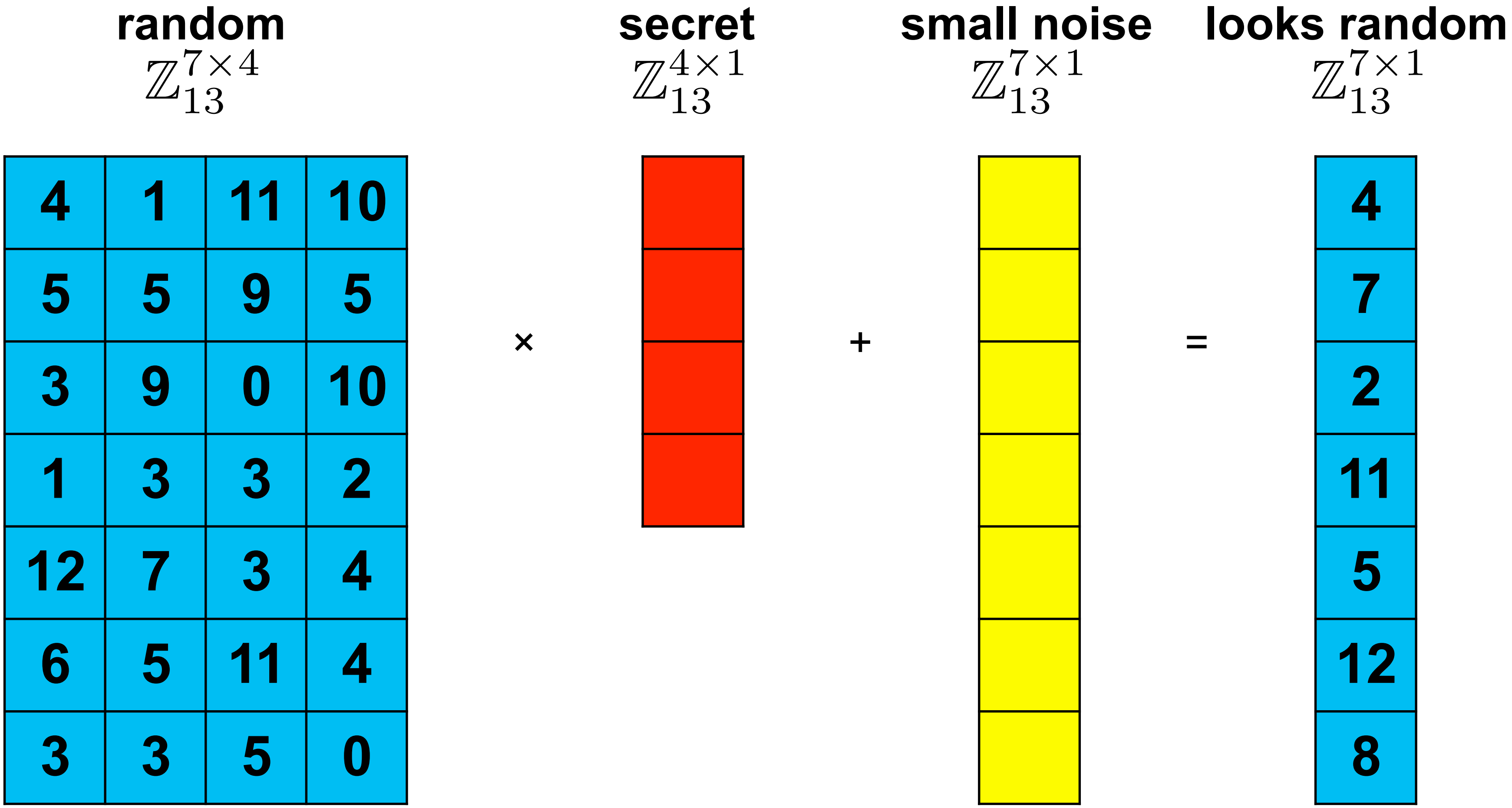
Easily solved using Gaussian elimination (Linear Algebra 101)

Linear system problem: given **blue**, find **red**

Learning with errors problem

$$\overset{\substack{\textbf{random}\\ \mathbb{Z}_{13}^{7\times 4}}}{\begin{pmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{pmatrix}} \times \overset{\substack{\textbf{secret}\\ \mathbb{Z}_{13}^{4\times 1}}}{\begin{pmatrix} 6 \\ 9 \\ 11 \\ 11 \end{pmatrix}} + \overset{\substack{\textbf{small noise}\\ \mathbb{Z}_{13}^{7\times 1}}}{\begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}} = \overset{\substack{\textbf{looks random}\\ \mathbb{Z}_{13}^{7\times 1}}}{\begin{pmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{pmatrix}}$$

Learning with errors problem



$$\begin{array}{ccccccc} \text{random} & & \text{secret} & & \text{small noise} & & \text{looks random} \\ \mathbb{Z}_{13}^{7\times 4} & & \mathbb{Z}_{13}^{4\times 1} & & \mathbb{Z}_{13}^{7\times 1} & & \mathbb{Z}_{13}^{7\times 1} \\ \begin{pmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{pmatrix} & \times & \begin{pmatrix} \\ \\ \\ \end{pmatrix} & + & \begin{pmatrix} \\ \\ \\ \\ \\ \\ \end{pmatrix} & = & \begin{pmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{pmatrix} \end{array}$$

LWE problem: given **blue**, find **red**

Toy example versus real-world example

$\mathbb{Z}_{13}^{7\times 4}$

4	1	11	10
5	5	9	5
3	9	0	10
1	3	3	2
12	7	3	4
6	5	11	4
3	3	5	0

$\mathbb{Z}_{4093}^{640\times 256}$

256

640

2738	3842	3345	2979	…
2896	595	3607		
377	1575			
2760				
…				

640 × 256 × 12 bits = **245 KiB**

Ring learning with errors problem

random

$\mathbb{Z}_{13}^{7 \times 4}$

4	1	11	10
10	4	1	11
11	10	4	1
1	11	10	4
4	1	11	10
10	4	1	11
11	10	4	1

Each row is the cyclic shift of the row above

Ring learning with errors problem

random
$\mathbb{Z}_{13}^{7\times 4}$

4	1	11	10
3	4	1	11
2	3	4	1
12	2	3	4
9	12	2	3
10	9	12	2
11	10	9	12

Each row is the cyclic shift of the row above

…

with a special wrapping rule: *x* wraps to –*x* mod 13.

Ring learning with errors problem

random

$\mathbb{Z}_{13}^{7\times 4}$

4	**1**	**11**	**10**

Each row is the cyclic shift of the row above

…

with a special wrapping rule: *x* wraps to –*x* mod 13.

So I only need to tell you the first row.

Ring learning with errors problem

$\mathbb{Z}_{13}[x]/\langle x^4+1\rangle$

$4 + 1x + 11x^2 + 10x^3$ random

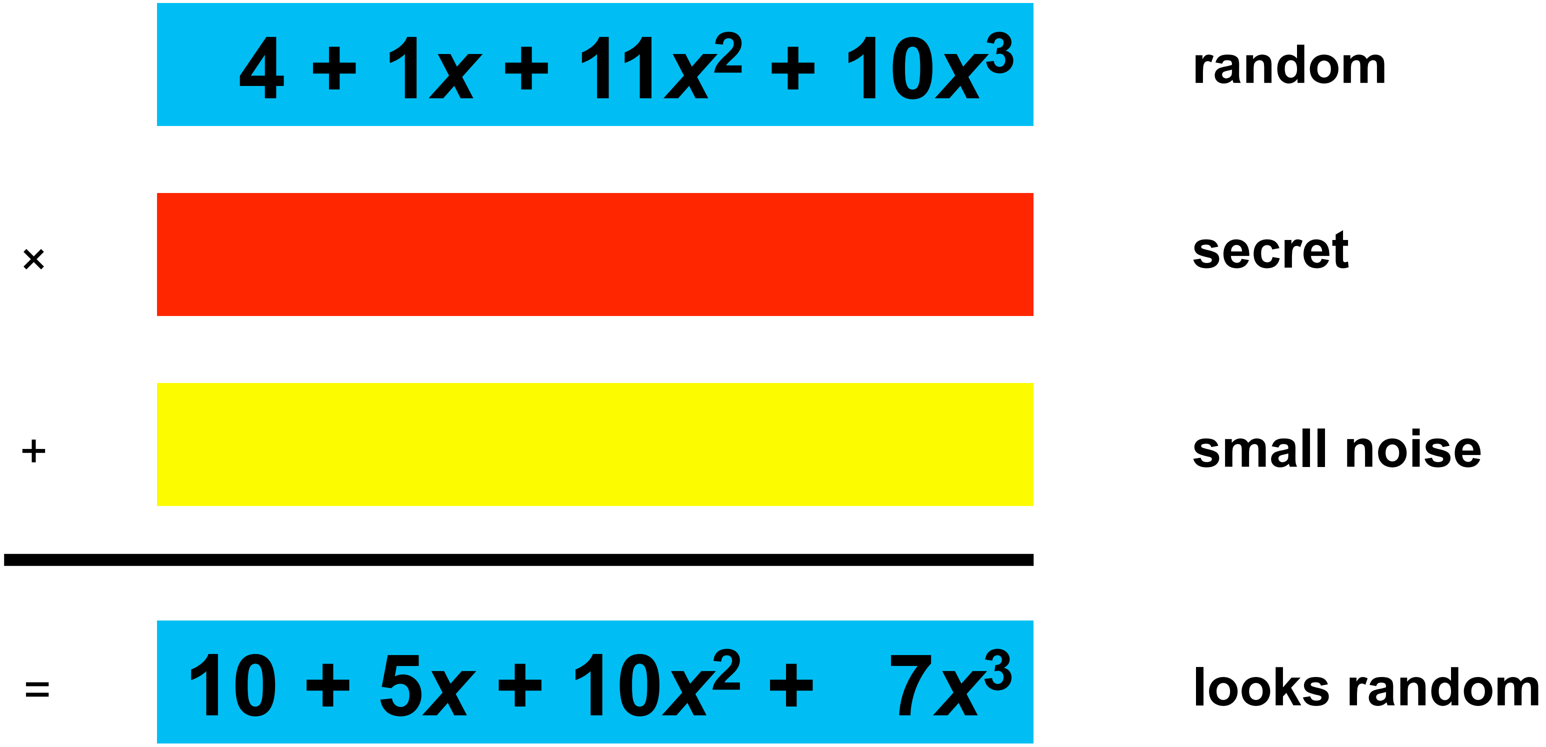
× $6 + 9x + 11x^2 + 11x^3$ secret

+ $0 - 1x + 1x^2 + 1x^3$ small noise

= $10 + 5x + 10x^2 + 7x^3$ looks random

Ring learning with errors problem

$$\mathbb{Z}_{13}[x]/\langle x^4+1\rangle$$



$4 + 1x + 11x^2 + 10x^3$ random

× secret

+ small noise

= $10 + 5x + 10x^2 + 7x^3$ looks random

Ring-LWE problem: given **blue**, find **red**

Ring learning with errors problem

$$\mathbb{Z}_{13}[x]/\langle x^4+1\rangle$$

For 128-bit security, need larger polynomials with larger coefficients.

$$\mathbb{Z}_{2^{32}-1}[x]/\langle x^{1024}+1\rangle$$

1024 × 32 bits = **4 KiB**

Ring-LWE problem: given **blue**, find **red**

Ring-LWE-DH key agreement (unauthenticated)

- Reformulation of Peikert's R-LWE KEM (*PQCrypto 2014*)

public: "big" a in $R_q = \mathbf{Z}_q[x]/(x^n+1)$

Alice

secret:
random "small" s, e in R_q

Bob

secret:
random "small" s', e' in R_q

$$\xrightarrow{\quad b = a \bullet s + e \quad}$$

$$\xleftarrow{\quad b' = a \bullet s' + e' \quad}$$

shared secret:
$s \bullet b' \approx s \bullet a \bullet s'$

shared secret:
$b \bullet s' \approx s \bullet a \bullet s'$

These are only approximately equal => need rounding

Ring-LWE-DH key agreement (unauthenticated)

- Reformulation of Peikert's R-LWE KEM (*PQCrypto 2014*)

Alice

secret:
randon

s', e' in R_q

Secure if decision ring learning with errors problem is hard.

Decision ring-LWE is hard if a related lattice shortest vector problem is hard.

shared secret:
$s \bullet b' \approx s \bullet a \bullet s'$

shared secret:
$b \bullet s' \approx s \bullet a \bullet s'$

These are only approximately equal => need rounding

Integration into TLS

New ciphersuite: **`TLS-RLWE-SIG-AES-GCM-SHA256`**

- RSA / ECDSA signatures for authentication
- Ring-LWE-DH for key exchange
- AES for authenticated encryption

Security

- Model: authenticated and confidential channel establishment (ACCE) (Jager et al., *Crypto 2012*)
- Theorem: signed ring-LWE ciphersuite is ACCE-secure if underlying primitives (signatures, ring-LWE, authenticated encryption) are secure
  - Interesting technical detail for ACCE provable security people: need to move server's signature to end of TLS handshake because oracle-DH assumptions don't hold for ring-LWE

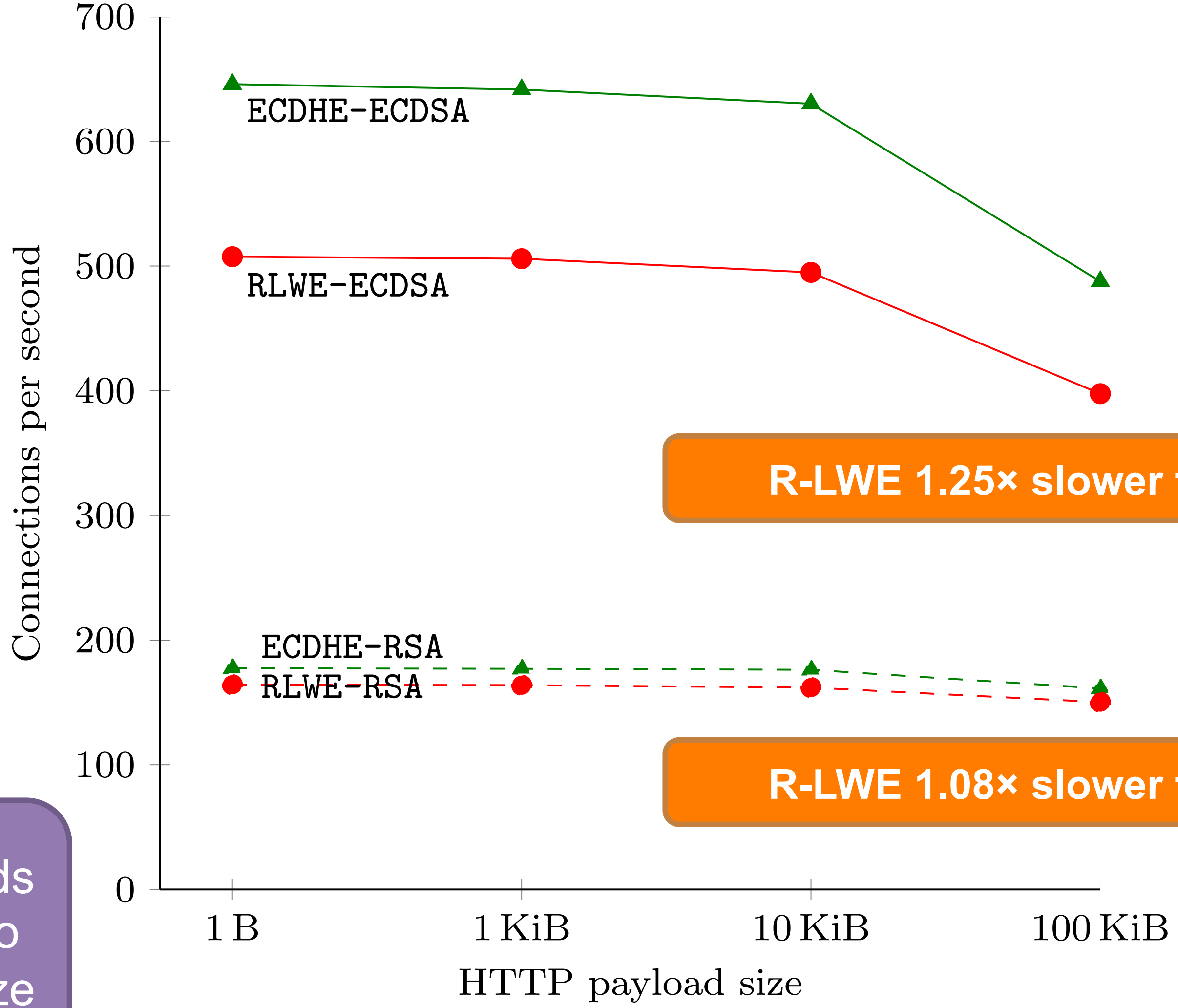
Performance – standalone

Operation	Client	Server
R-LWE key generation	0.9ms	0.9ms
R-LWE Alice	0.5ms	
R-LWE Bob		0.1ms
R-LWE total runtime	**1.4ms**	**1.0ms**
ECDH nistp256 (OpenSSL)	**0.8ms**	**0.8ms**

R-LWE 1.75× slower than ECDH

constant-time implementation
Intel Core i5 (4570R), 4 cores @ 2.7 GHz
llvm 5.1 (clang 503.0.30) –O3
OpenSSL 1.0.1f

Performance – in TLS

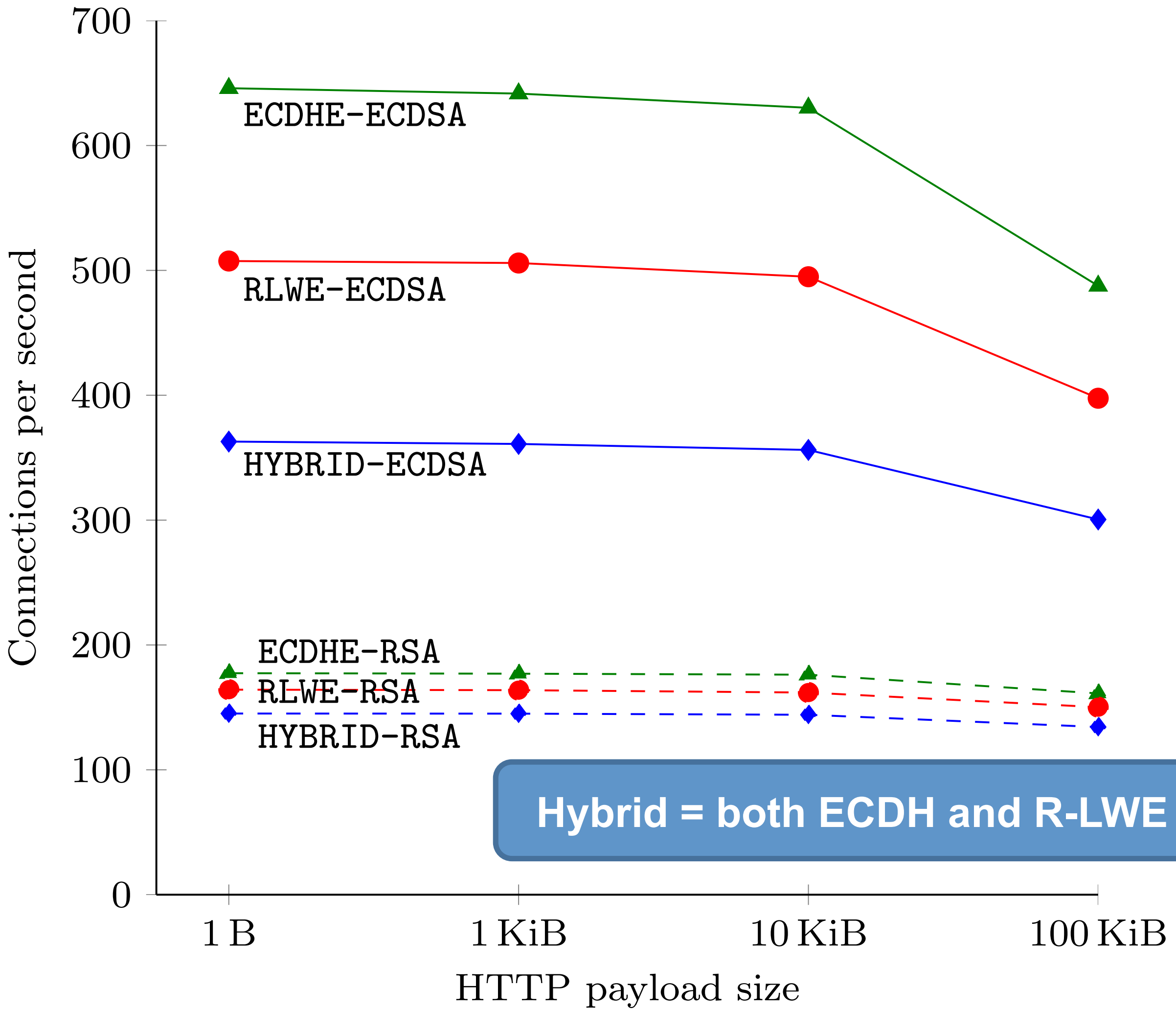


R-LWE 1.25× slower than ECDH

R-LWE 1.08× slower than ECDH

Ring-LWE adds about 8 KiB to handshake size

Performance – in TLS



Hybrid = both ECDH and R-LWE key exchange

Answers to questions

Ring-LWE ciphersuite with traditional signatures:

- Key sizes: not too bad (8 KiB overhead)
- Performance: small overhead (1.1–1.25×) within TLS.
- Integration into TLS: requires reordering messages, but otherwise okay.

Caveat: lattice-based assumptions less studied, algorithms solving ring-LWE may improve, security parameter estimation may evolve.

Future work:

- better attacks
- ring-LWE performance improvements:
  - assembly, alternative FFT, better sampling, …
- other post-quantum key exchange algorithms
- post-quantum authentication

Links

The paper

- http://eprint.iacr.org/2014/599

Magma code:

- http://research.microsoft.com/en-US/downloads/6bd592d7-cf8a-4445-b736-1fc39885dc6e/default.aspx

Standalone C implementation

- https://github.com/dstebila/rlwekex

Integration into OpenSSL

- https://github.com/dstebila/openssl-rlwekex